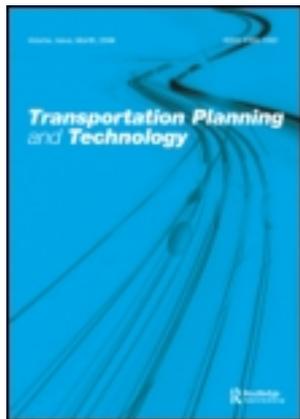


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A FUZZY QUEUING MODEL FOR REAL-TIME, ADAPTIVE PREDICTION OF INCIDENT DELAY FOR ATMS/ATIS

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This paper presents a fuzzy queuing model that can be used to predict the possible delay that a vehicle will experience at an incident location based on real-time information on current queuing conditions, traffic arrivals, lane closings. Compared to most existing methods, the proposed model is unique in three aspects. First, it explicitly accounts for uncertainties involved in all influencing factors and thus allows easy incorporation of imprecise and vague information typically available in this type of prediction environment. Second, the model is adaptive in the way that it allows continuous update of estimates as new information is made available. Third, delays obtained from the model are fuzzy numbers that can be conveniently mapped to linguistic terms for use in systems such as changeable message signs (CMS). A case study is presented to demonstrate the application of the proposed model in facilitating the composition of location-dependent delay messages for CMS.

Keywords: Incident delay, Queuing model, Fuzzy set, Link travel time, ATIS, ATMS

1. INTRODUCTION

Provision of timely and reliable information on traffic incidents and subsequently induced congestion is a critical ability to the successful

deployment of many envisioned Advanced Traffic Management and Information Systems (ATMIS). Ideally, anticipated and quantitative information such as time-dependent delay caused by an incident should be estimated and provided to drivers to maximize the effect of information provision. Procurement of such information is however not a trivial task because of the complex interactions among various factors such as incident location and severity, incident response capacity, demand fluctuation and diverse driver responses to information. Moreover, most of these factors are subject to high uncertainty and information available to quantify them is often incomplete and subjective in nature. Consequently, provision of crisp values of expected delays to drivers through systems such as changeable message signs (CMS) would inevitably lower drivers' trust in the accuracy of the provided information because the actual delays they would experience will be either larger or smaller than what were suggested. This underlying dilemma has become a major reason for many traffic management authorities to opt for less effective, but more credible alternatives such as providing qualitative information only. The goal of this paper is to demonstrate that, with an appropriate delay prediction model, it is possible to resolve this dilemma.

Traditionally, incident delay is estimated using a deterministic queuing model that assumes that the traffic arrival rate, capacity reduction and incident duration can be identified exactly. This approach may be adequate for 'after' evaluation where the information on the traffic volume and incident situation is readily available. However, it is inappropriate for prediction of incident delay in real-time applications because the only information typically available is some linguistic descriptions on the current status of the incident (removed or not/lane closures/current queue), possible traffic diversion (traffic volume) and the time required to remove the incident (incident duration). Such linguistic descriptions inevitably involve uncertainties such as imprecision and vagueness or fuzziness. Ignorance of these inherent uncertainties may result in a biased prediction of the incident delay as well as a loss of subjective information on the pattern of the incident delay (e.g. variation of incident delay) that may be significant for many transportation applications [1,2]. The objective of the present research was to develop an incident delay prediction model that can explicitly consider the uncertainties involved in traffic demand, capacity, incident duration and current queuing status.

Most of the existing incident delay estimation methods focus on the

estimation of total delay caused by incidents. Examples include Chow [3], the Highway Capacity Manual [4], Wirasinghe [5], Morales [6] and Al-Deek *et al.* [7]. These methods are intended only for 'after' incident evaluation and information on traffic conditions and incident situation is assumed known. Messer *et al.* [8] developed a method for predicting the travel time required to traverse a freeway segment with incident congestion. The model was developed based on the shock wave theory for use in the operations and control of CMS. In their research, it was assumed that all of the inputs were known a priori and the model is therefore deterministic. Recently, Fu and Rilett [9] developed a stochastic incident delay model in which the incident duration is assumed to be a random variable with known distribution and all other parameters are assumed deterministic. It should be noted that it becomes mathematically intractable to develop similar types of prediction models if more than one parameter is needed for consideration as random variables.

With the rapid development of Intelligent Transportation Systems (ITS) field in the past decade, various link travel time estimation and prediction methods have been proposed for demonstration ITS projects and simulation studies. Hoffman and Janko [10] developed a link travel time estimation and prediction method that has been used in the ALI-SCOUT system. In their approach the link travel time is predicted by scaling the historical travel time based on current detected link travel time. Koutsopoulos and Xu [11] presented an approach based on information discounting theory as an attempt to improve Hoffman's model. In the ADVANCE project [12], the proposed link travel time estimation method treats incident-absent situations and incident-present situations separately, in which historical link travel time is used for calculating vehicle routes. It is important to note that all of these methods are fundamentally heuristic and do not fully make use of information on the status of the incident such as lane closure, possible incident duration and traffic volume. Therefore, techniques that simply use a scaling factor may result in over-estimation or underestimation of the actual link travel time.

Many simulation studies have been conducted to evaluate the potential benefits of ATMS/ATIS. One of the essential components of these simulation models is the link travel time model that is used to estimate the link travel time for use in route calculation. Koutsopoulos and Yablonski [13] presented a theoretical link travel time estimation model in which the incident delay was estimated by a deterministic model. Although the incident and their attributes (reduced capacity and

incident duration) were randomly generated, this information was assumed to be known and was used directly for identifying optimal routes. Al-Deek and Kanafani [14] evaluated the benefits of an ATIS application called in-vehicle route guidance systems (RGS) specifically in the case of incident congestion. In their model, incident duration was also assumed to be deterministic, and a deterministic queuing model was used to estimate the queuing delay. All these simulation applications assumed an advanced knowledge of the incident situation and then 'guide' the vehicles based on this information. However, in a real-time operation, the evolution of incident situation can not be predicted exactly and the use of such 'extra' information may result in over-estimation of the benefits from an ATMS/ATIS project.

The imprecision of travel time data and its implication to various related decision-making problems have been studied by several other researchers. Teodorovic and Kikuchi [15] applied fuzzy inference technique to address the route choice problem with fuzzy link travel times. As an extension, Lotan and Koutsopoulos [16] proposed a framework for modeling route choice behaviour under provision of real-time traffic information. Kikuchi and Donnelly [2] conducted a study on a specific dial-a-ride vehicle routing and scheduling problem where the origin–destination (O–D) travel time and the desired time of vehicle stop requested by customers are modeled as fuzzy numbers. These studies however did not provide any specifics on how fuzzy travel times can be estimated, especially during incident conditions. Akiyama and Yamanishi [17] proposed a method to transform forecast travel time data (crisp) to practical information (fuzzy numbers) to service the end users (drivers). In their paper, it is not clear whether or not various fuzzy information that may be available in a real-time prediction environment is actually used to calculate the travel time value.

This paper proposes a framework based on fuzzy set theory to model the evolution of incident congestion or queue development. The rationale behind this approach is that information typically available under incident conditions is often in the form of linguistic descriptions characterized by imprecision and vagueness. The paper first describes various uncertainties involved during incident conditions and how they can be systematically modeled on the basis of fuzzy set theory. A fuzzy queuing model is subsequently presented for predicting the possible delay that a vehicle will experience at an incident location based on real-time information on current queuing conditions, future

traffic arrivals, capacity reductions and the vehicle's arrival time. Sensitivity analyses of the estimation error by a deterministic model as a function of the fuzziness of various input variables are finally performed. A case study is then presented to demonstrate the application of the proposed model in composing location-dependent delay messages for CMS.

2. A FUZZY INCIDENT DELAY MODEL

2.1. Problem Statement

Consider a hypothetical case that an incident has occurred on a high-volume road section, which has caused queue and congestion due to reduction of capacity on this road section. As a result, it can be expected that the queue and congestion will begin to buildup from the location of incident and any vehicle entering this road section may experience delay caused by the congestion. The problem is to provide a prediction, at current moment, of the incident delay that a vehicle may experience if it were to enter this road section at a given time in future.

Delay that a vehicle will experience as a result of an incident depends on many factors including incident severity (capacity reduction), incident duration, traffic volume and the time when the vehicle arrives at the incident location. In a practical situation, each of these factors is subject to uncertainty. It would be a matter of a simple application of deterministic queuing theory or shock wave theory if we could predict the exact value of these factors. However, in reality, most of these factors are subject to a certain level of uncertainty and the information that may be available for estimating these factors is commonly imprecise or vague. The following sections discuss the sources of these uncertainties and how they can be modeled as fuzzy numbers.

2.1.1. Traffic Arrival Rate

Under normal traffic condition, the traffic arrival rate is usually stable and can be fairly accurately estimated based on historical traffic counts. However, during incident conditions, the prediction of traffic arrivals is no longer a trivial task because some drivers may have been informed of the incident occurrence and decide to divert to other

routes. How much traffic will divert and at what rate will depend on many factors such as traffic information coverage, drivers' acceptance of the provided information and local network conditions. Currently, there is no dependable model available that can be used to capture these complexities and provide an accurate prediction of the dynamic traffic conditions. However, it can be reasonably expected that an approximate estimation of the traffic diversion may be available. For example, an experienced traffic manager may be able to give such estimation as 'About 20–30% of the traffic will divert to avoid the incident congestion', or 'A majority of the drivers will still use the route even though one lane is closed due to the incident'. These linguistic terms can be properly modeled by a fuzzy set or a fuzzy number.

2.1.2. Incident Duration

The time taken to remove an incident and recover the road capacity, or incident duration, is another key piece of information needed for predicting the incident delay. It has been observed that incident duration usually has a large variation depending on incident severity and location, traffic conditions and the availability of incident management [18,19]. For example, Giuliano [18] showed that the mean duration is about 37 min with a standard deviation of 30 min while Cohen and Nouveliere [19] indicated that the mean duration is 26 min with a standard deviation of 23 min. Therefore, it is nearly impossible to give a precise prediction of the incident duration even when there is a large amount of historical data available. However, it is not unusual for an experienced incident response team or highway police to give an estimation of the duration after they know the incident situation and location. For example, they may provide statements such as 'it would take about 30 to 40 min to remove the debris', 'it will take at least one hour' or 'It shouldn't take longer than two hours'. Such information presented in linguistic terms is commonly imprecise or vague and can be adequately represented using fuzzy numbers.

2.1.3. Current Queue

There is usually a time lag between the current time (the time to make a prediction) and the incident occurrence. Therefore, it is likely that a queue has formed at the incident location. The current queue can be

estimated based on information such as the elapsed time from incident occurrence, traffic arrivals and reduced capacity. However, it is more likely that it can be directly obtained from various information sources such as observers, police or a special incident response team. This information is usually a linguistic description on the queuing status (e.g. 'the queue is about to backup to the 12th street'). It should be noted that in most cases, these descriptions often describe the current queue reach instead of the queue length. However, these two variables can be considered as the same before the incident is removed. For the same reason as for the previous parameters, it can be nicely represented using a fuzzy set.

2.1.4. Capacity

The capacity under normal traffic condition can be considered as constant for a given road section and estimated based on HCM [4]. During incident conditions, it has been observed that the departure rate from the queue (or capacity during the incident) varies significantly because of the stop-and-go process and 'gawkers block'. The actual value can be as low as 1500 to as high as 2000 pcu/h/lane, depending on the local conditions and driver behavior. It has also been observed that the laneblocking incidents have more than a proportional impact on capacity. For example, Urbanek and Rogers [20] indicated that the blockage of a single lane on a three-lane facility reduced freeway capacity by 40–50% (instead of 33% based on space reduction). Accordingly, it is desirable to use a fuzzy number to model the reduced capacity during incident.

2.1.5. Vehicle Arrival Time

The incident delay that a vehicle may experience also depends on when the vehicle will arrive at the incident location. For example, if a Traffic Information Center (TIC) is to provide information to a vehicle currently at a known location, the prediction of incident delay also requires the estimation of travel time from the current location to the incident location. There is no doubt that this travel time involves uncertainty caused by many factors such as variation of traffic demands, traffic control and driving conditions. Although this travel time can be modeled as a random variable, the underlying distribution may not follow a popular mathematical distribution such as normal, log-

normal or Beta distribution. Therefore, it is also appropriate to use a fuzzy number to represent the travel time or arrival time at the incident location.

2.2. Fuzzy Incident Delay

This section is to discuss how the incident delay can be deduced when some of the input parameters are modeled as fuzzy numbers. The methodology is applying a traditional deterministic queuing model with input parameters systematically represented by fuzzy numbers. The functional relationship between the fuzzy incident delay and the input variables are established based on arithmetic operations of fuzzy numbers using α -cut concept [21].

To simplify the model development, this paper assumes that the traffic arrival rate and the discharge rate during an incident do not change with time over the time period when the incident impact prevails; it is also assumed that the road section or link is long enough so that there will be no spill-back to the upstream link. It should be noted that although the model does not explicitly consider the situation that the incident has been removed and the original capacity has been restored, it is indeed a special case of the developed model by assuming the incident duration equal to a single value of zero.

The parameters used in the following discussion are defined as follows.

The values of these parameters are known and used as basic input:

V = a fuzzy number representing the average traffic arrival rate over the time period when the incident impact prevails (pcu/h). The membership function of V is assumed to be known and denoted as $\mu V(x)$. The interval of confidence for the level of presumption α , $\alpha \in [0,1]$ is denoted as $V\alpha = [v_1^{(\alpha)}, v_2^{(\alpha)}]$ where $v_1^{(\alpha)}$ and $v_2^{(\alpha)}$ can be calculated based on $\mu V(x)$;

Q = a fuzzy number indicating the current number of queuing vehicles with a known membership function denoted as $\mu Q(x)$. Its interval of confidence of level α , $\alpha \in [0,1]$ is denoted as $Q\alpha = [q_1^{(\alpha)}, q_2^{(\alpha)}]$ where $q_1^{(\alpha)}$ and $q_2^{(\alpha)}$ can be calculated based on $\mu Q(x)$;

s = a deterministic crisp variable denoting the link capacity after the incident is removed (pcu/h);

C = a fuzzy number designating the reduced link capacity caused by

the incident (pcu/h). The membership function of C is assumed to be known and denoted as $\mu C(x)$. The interval of confidence for the level of presumption α , $\alpha \in [0,1]$ is denoted as $C\alpha = [c_1^{(\alpha)}, c_2^{(\alpha)}]$ where $c_1^{(\alpha)}$ and $c_2^{(\alpha)}$ can be calculated based on $\mu C(x)$;

L = a fuzzy number representing the incident duration, or the time required to remove the incident (from current time). The membership function of L is assumed to be known and denoted as $\mu L(x)$. The interval of confidence for the level of presumption α , $\alpha \in [0,1]$ is denoted as $L\alpha = [l_1^{(\alpha)}, l_2^{(\alpha)}]$ where $l_1^{(\alpha)}$ and $l_2^{(\alpha)}$ can be calculated based on $\mu L(x)$;

T = a fuzzy number representing the time when a vehicle arrives at the incident location (from current time). The membership function of T is assumed to be known and denoted as $\mu T(x)$. The interval of confidence for the level of presumption α , $\alpha \in [0,1]$ is denoted as $T\alpha = [t_1^{(\alpha)}, t_2^{(\alpha)}]$ where $t_1^{(\alpha)}$ and $t_2^{(\alpha)}$ can be calculated based on $\mu T(x)$.

Figure 1 shows the α -cut representation of a fuzzy queuing model where the cumulative traffic arrivals are represented by two straight lines with rates of $v_1^{(\alpha)}$ and $v_2^{(\alpha)}$ respectively. The α -cut of the capacity during incident condition are represented by two straight lines with rates of $c_1^{(\alpha)}$ and $c_2^{(\alpha)}$, which intersect with lines representing recovered capacity, s , at point A and B. The lower bound of arrivals and upper bound of discharges intersect at point C while the upper bound of arrivals and lower bound of discharges meet at point D. The related parameters are defined as follows.

N_A = cumulative number of vehicles discharged at the end of incident duration $l_1^{(\alpha)}$ with discharge rate (reduced capacity) of $c_2^{(\alpha)}$ (point A in Fig. 1). It can be determined by Eq. (1):

$$N_A = c_2^{(\alpha)} \lambda^{(\alpha)} \quad (1)$$

N_B = cumulative number of vehicles discharged at the end of incident duration $l_2^{(\alpha)}$ with a discharge rate (reduced capacity) of $c_1^{(\alpha)}$ (point B in Fig. 1). It can be determined by Eq. (2):

$$N_B = c_1^{(\alpha)} \lambda^{(\alpha)} \quad (2)$$

N_C = cumulative number of vehicles arrived at the end of incident clearance corresponding to arrival rate $v_1^{(\alpha)}$, incident duration of $l_1^{(\alpha)}$ and discharge rate of $c_2^{(\alpha)}$ (point C in Fig. 1). It can be determined by Eq. (3):

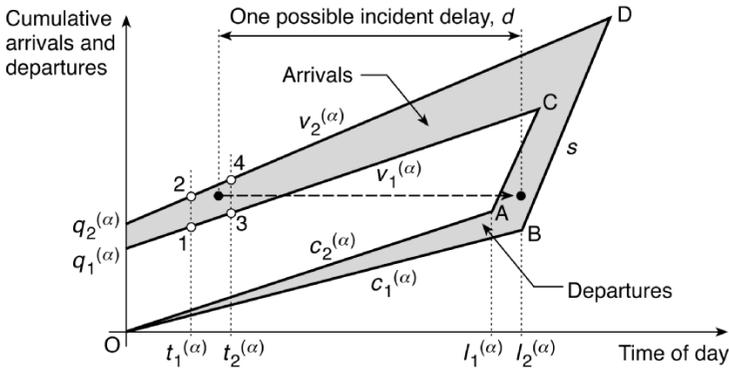


FIGURE 1 An α -cut representation of a fuzzy incident delay queuing model.

$$N_C = \frac{q_1^{(\alpha)} \cdot s + v_1^{(\alpha)} \lambda_1^{(\alpha)} \cdot (s - c_2^{(\alpha)})}{s - v_1^{(\alpha)}} \quad (3)$$

N_D = cumulative number of vehicles arrived at the end of incident clearance corresponding to arrival rate $v_2^{(\alpha)}$, incident duration of $l_2^{(\alpha)}$ and discharge rate of $c_1^{(\alpha)}$ (point D in Fig. 1). It can be determined by Eq. (4):

$$N_D = \frac{q_2^{(\alpha)} \cdot s + v_2^{(\alpha)} \lambda_2^{(\alpha)} \cdot (s - c_1^{(\alpha)})}{s - v_2^{(\alpha)}} \quad (4)$$

For a vehicle arriving at the time interval, $[t_1^{(\alpha)}, t_2^{(\alpha)}]$, it may join a possible queue as represented by a point within the polygon 1–2–3–4, as shown in Fig. 1. The incident delay that could possibly be experienced by this vehicle is the horizontal distance (in time units) from a point within the polygon to a possible discharge point bounded by the lines O-A-C and O-B-D. The α -cut of the incident delay can therefore be determined if the lower and upper bounds of the delay can be estimated.

The lower bound of the delay is the minimum horizontal distance from the polygon 1–2–3–4 to the line O-A-C while the upper bound is the maximum horizontal distance from the polygon to the line O-B-D. Because the ending zone, as boxed by lines O-A-C and O-B-D, and two horizontal lines from point 1 and 4, is in most cases a polygon with four vertices, the minimum and maximum horizontal distances can be identified by examining the minimum and maximum delays that

a vehicle would experience if it joints queue at point 1, 2, 3 and 4 (Fig. 1).

Denote the coordinates associated with the four corner points 1, 2, 3 and 4 shown in Fig. 1 as $\{t_k, n_k; k = 1,2,3,4\}$, where

$$\begin{aligned} t_1 &= t_1^{(\alpha)} ; n_1 = q_1^{(\alpha)} + v_1^{(\alpha)} . t_1^{(\alpha)} \\ t_2 &= t_1^{(\alpha)} ; n_2 = q_2^{(\alpha)} + v_2^{(\alpha)} . t_1^{(\alpha)} \\ t_3 &= t_2^{(\alpha)} ; n_3 = q_1^{(\alpha)} + v_1^{(\alpha)} . t_2^{(\alpha)} \\ t_4 &= t_2^{(\alpha)} ; n_4 = q_2^{(\alpha)} + v_2^{(\alpha)} . t_2^{(\alpha)} \end{aligned}$$

The minimum delay and the maximum delay for a given point k , denoted as d_1^k and d_2^k respectively, can be determined using the following equations:

$$d_1^k = \begin{cases} 0 & \text{if } n_k \geq N_C \\ \frac{n_k}{c_2^{(\alpha)}} - t_k & \text{if } n_k \leq N_A \\ \frac{n_k - \lambda_1^{(\alpha)} . c_2^{(\alpha)}}{s} + \lambda^{(\alpha)} - t_k & \text{if } N_A < n_k < N_C \end{cases} \quad (6)$$

$$d_2^k = \begin{cases} 0 & \text{if } n_k \geq N_D \\ \frac{n_k}{c_2^{(\alpha)}} - t_k & \text{if } n_k \leq N_B \\ \frac{n_k - \lambda_2^{(\alpha)} . c_2^{(\alpha)}}{s} + \lambda^{(\alpha)} - t_k & \text{if } N_B < n_k < N_C \end{cases} \quad (7)$$

The delay interval is therefore

$$D\alpha = [d_1^{(\alpha)}, d_2^{(\alpha)}] \quad (8)$$

where $d_1^{(\alpha)} = \min_{k=1,2} \{d_1^k\}$

$$d_2^{(\alpha)} = \max_{k=3,4} \{d_2^k, d_0\}$$

2.3. Expected Incident Delay

The fuzzy incident delay model developed in the previous section depicts a complete pattern of the imprecision or fuzziness of incident delay and thus provides the necessary information for any applications in which the uncertainty of incident delay is explicitly considered in

decision-making process or information provision. However, there is often a situation in which a single represented value (or expected value) is finally used as input to other decision-making processes. For example, an RGS commonly uses a single value of link travel time (including delay) to identify the optimal routes in a traffic network. The expected value of a fuzzy set can be calculated using the centroid of gravity technique. For a fuzzy set A with membership function $\mu_T(x)$, the expected value (z_A) is defined as:

$$z_A = \frac{\int x \mu_A(x) dx}{\int \mu_A(x) dx} \quad (9)$$

Since Eq. (8) gives only the interval of confidence at level of presumption α instead of an explicit membership function of the incident delay, an approximation scheme has to be used to calculate the integration involved in Eq. (9). Assume that M levels of α -cuts represented by vector $\{\alpha_0 = 0.0, \alpha_1, \alpha_2, \dots, \alpha_M = 1.0\}$ are used for calculating fuzzy incident delay. For an α -cut set of level α_i , the confidence of interval can be calculated based on Eq. (8). Given all the confidence of intervals, the centroid z_D of the fuzzy incident delay can then be approximated by:

$$z_D = \frac{\sum_{i=0}^{M-1} \left\{ (\alpha_{i+1} + \alpha_i) \cdot (d_2^{(\alpha_{i+1})} - d_1^{(\alpha_{i+1})} + d_2^{(\alpha_i)} - d_1^{(\alpha_i)}) \cdot z_i \right\}}{\sum_{i=0}^{M-1} \left\{ (\alpha_{i+1} + \alpha_i) \cdot (d_2^{(\alpha_{i+1})} - d_1^{(\alpha_{i+1})} + d_2^{(\alpha_i)} - d_1^{(\alpha_i)}) \cdot z_i \right\}} \quad (10)$$

$$\text{where } z_i = \frac{d_1^{(\alpha_i)} + d_1^{(\alpha_{i+1})} + d_2^{(\alpha_i)} + d_2^{(\alpha_{i+1})}}{4}$$

3. SENSITIVITY ANALYSIS: A COMPARISON TO DETERMINISTIC MODEL

As discussed in section 1, incident delay is traditionally estimated using a deterministic queuing model which assumes that the attributes of an incident (capacity reduction, duration) are known or can be predicted exactly. The incident delay based on deterministic queuing model can be calculated by Eq. (11), where the same notation used for the fuzzy queuing model are utilized, except they should be interpreted as deterministic variables:

$$d = \begin{cases} 0 & \text{if } T \geq T_2 \\ \frac{Q + T.(V - C)}{C} & \text{if } T < T_1 \\ \frac{Q + T.(V - S) + L.(S - C)}{s} & \text{if } T_1 < T < T_2 \end{cases} \quad (11)$$

$$\text{where } T_1 = \frac{C.L - Q}{V} \text{ and } T_2 = \frac{Q + L.(S - C)}{S - V}$$

Apart from the fact that the deterministic model does not provide information on the uncertainty of incident delay and is incapable of incorporating various fuzzy information, it may also generate a biased estimate of the expected incident. The following section provides a numerical example to demonstrate the estimation bias of the deterministic model and its sensitivity to the imprecision in input parameters including incident duration, capacity, traffic arrival rate and initial queue.

Consider the case that an accident has been detected on a three-lane freeway segment. The traffic management center needs to predict the possible delay for a vehicle that were to enter that freeway segment. It is assumed that the traffic arrival rate, incident duration and capacity during incident can be modeled as symmetrical fuzzy trapezoidal numbers (TrFN) with known membership functions. Note that for a TrFN, the membership function can be defined by four parameters, $\{b_1, b_2, b_3, b_4\}$. The data used for analysis as a base case are summarized in Table I, where the expected values are given for use to estimate the delay by a deterministic model. In addition, a measure called *relative divergence* is used to define the fuzziness of a fuzzy number [21]. For a symmetrical TrFN represented by $\{m - \delta_2, m - \delta_1, m + \delta_1, m + \delta_2\}$ (where m is the expected value), the relative divergence or the fuzziness can be determined by $(\delta_1 + \delta_2)/2m$.

Based on the given data, the fuzzy incident delay and the expected incident delay (Eqs (8) and (10)), as well as the incident delay estimated by the deterministic model (Eq. (11)) can be calculated for further analysis. The fuzzy incident delay is approximated using five levels of presumption with $\alpha = \{0; 0.2; 0.4; 0.6; 0.8; 1.0\}$.

Figure 2 shows a series of fuzzy incident delays corresponding to different vehicle arrival times when the incident duration is a fuzzy number and all other parameters are deterministic (taking the expected value given in Table I). As it would be expected, any earlier arrivals

TABLE I Example data for sensitivity analysis

<i>Input variables</i>	<i>Expected value</i>	<i>Fuzzy representation</i>	<i>Fuzziness</i>
Traffic volume (pcu/h)	4000	(3175, 3225, 4775, 4825)	20%
Capacity during incident (pcu/h)	3200	(2535, 2585, 3815, 3865)	20%
Initial Queue (pcu)	60	(45, 50, 69, 74)	20%
Incident duration* (minute)	10	(7, 9, 11, 13)	20%
Full capacity (pcu/h)	5400	(5400, 5400, 5400, 5400)	0%

(e.g. before 5 min from current time) will experience a definite amount of delay with very little imprecision involved (see Figs 2(a) and 2(b)). After that, the variation of the incident duration takes effect and the fuzziness of the predicted incident delay becomes significant. For example, for a vehicle arriving at the incident location in 10 min, its possible delay ranges from 56 to 202 s (Fig. 2(c)). Due to the possible removal of the incident, the expected value and the fuzziness of the incident delay for vehicles arriving after the expected incident duration decreases (Figs 2(d) and 2(e)). Finally, the incident queue and resulting congestion is cleared and vehicles will experience no delay at all (Fig. 2(f)).

A sensitivity analysis is conducted by assuming one of the input parameters is a fuzzy number and the rest take the deterministic values given in Table I. The variation of the fuzziness of the input variable is generated by increasing or decreasing the fuzziness around the base case number.

Figure 3 illustrates the expected delay that a vehicle may experience as a function of its arrival time at the incident location under different fuzziness of incident duration. It can be seen that the deterministic model would over-estimate the maximum expected incident delay while it gives under-estimation for vehicles with arrival time around the expected queue clearance time. For example, in the base case (20% fuzziness), if the vehicle arrives at the link 7 min after the incident, the deterministic model would over-estimate the expected incident delay by approximately 20%. The deterministic model predicted no delay for a vehicle arriving at 20 min while the fuzzy model predicted a delay of 20 s. The maximum estimation error increases proportionally with the imprecision of the incident duration. It should be noted that similar finding has been reported by Fu and Rilett [9] although their analysis was based on probabilistic theory.

As shown in Fig. 4, the fuzziness of the traffic volume has a significant impact on the prediction of the expected incident delay. Although the delay is slightly overestimated by the deterministic model, the major problem resulting from the deterministic treatment is the underestimation of the expected delay. For example, if the fuzziness of the arrival traffic volume is 20%, the fuzzy model shows that a vehicle may be delayed by the incident even arriving 40 min after the occurrence of the incident. The maximum under-estimated amount is approximately 50% of the maximum delay.

Figure 5 shows the presence of fuzziness in the capacity during

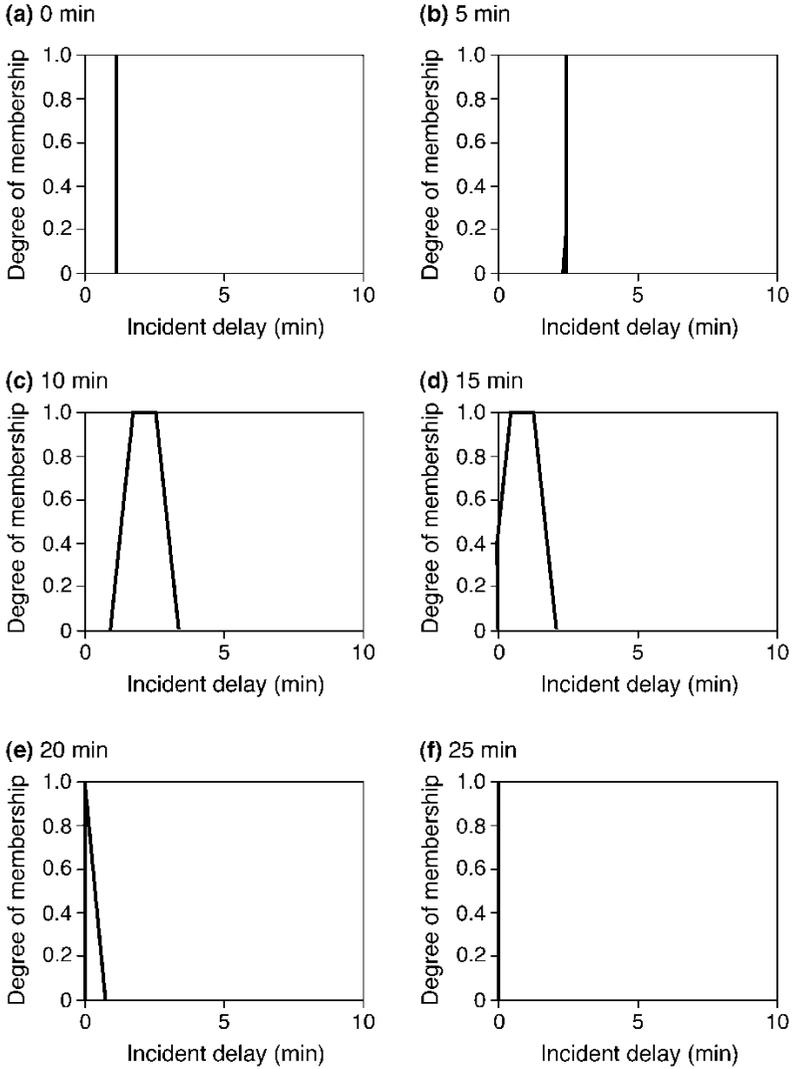


FIGURE 2 Fuzzy incident delay as a function of arrival time.

incident would mostly result in under-estimation of the expected delay. As the fuzziness of the incident capacity increases, the estimation error increases proportionally. However, it seems to be that the time period during which there is a nonzero expected delay is bounded (e.g. 25

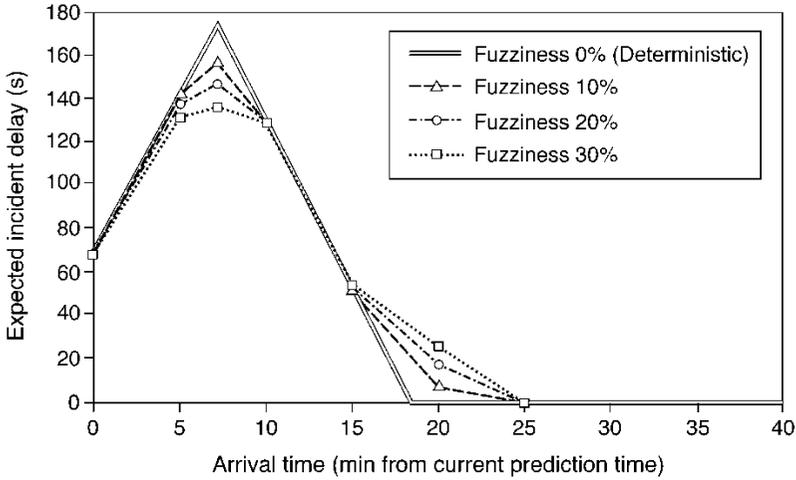


FIGURE 3 Relationship between expected incident delay and fuzziness of incident duration.

min in this example). The time when the maximum expected delay occurs tends to shift earlier as the fuzziness of the capacity increases.

Different from the previous parameters, the fuzziness of the initial queue has negligible impact on the prediction of the expected incident

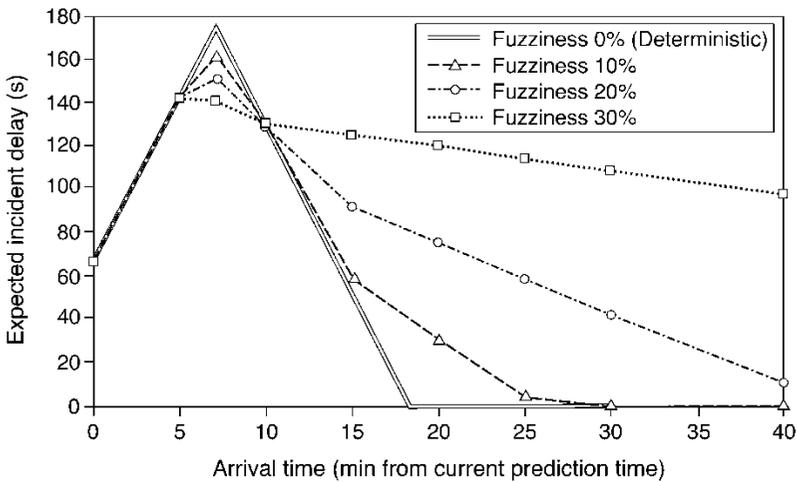


FIGURE 4 Relationship between expected incident delay and fuzziness of traffic arrival rate.

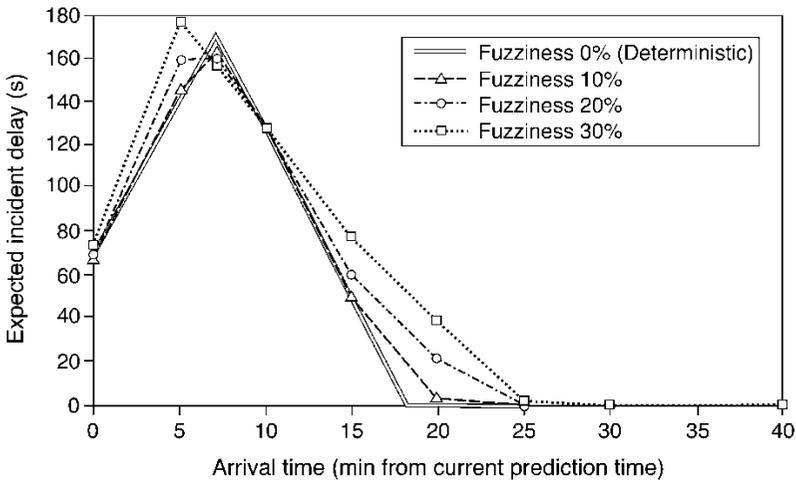


FIGURE 5 Relationship between expected incident delay and fuzziness of capacity during incident.

delay, as shown in Fig. 6. This implies that it may not be necessary to consider the imprecision of the current queue status when only the expected incident delay is to be estimated.

4. AN EXAMPLE APPLICATION

This section presents a hypothetical case to demonstrate the potential application of the proposed model. Consider the case that an accident was detected on a three-lane freeway section at 3:20 p.m., as shown in Fig. 2. Two CMS are available for the traffic management center (TMC) to post incident delay information. The messages are intended for drivers who are just in the view of one of the CMS. In order to determine what message should be displayed, the TMC needs to predict the possible delays that would be experienced by vehicles if they were to continue to travel on the freeway section instead of diverting to other routes. It is assumed that some incomplete pieces of information are available which permit the representation of the traffic arrival rate, incident duration, capacity during incident, and current queuing status as fuzzy trapezoidal numbers (TrFN) represented by $\{a, m, n, b\}$. The data used for analysis are summarized in Fig. 7. Two prediction scenarios are considered. Scenario 1 represents the estimation task at the time 3:20 p.m., that is, right at the time the incident

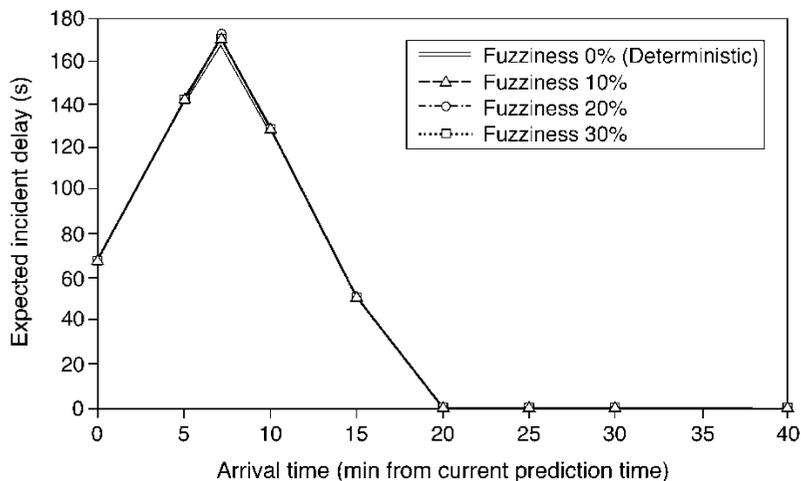


FIGURE 6 Relationship between expected incident delay and fuzziness of initial queue.

is detected while scenario 2 models the prediction task at 3:40 p.m., at which time the incident is expected to be removed soon. Note from the data that a larger value of current queue was used in scenario 2 as compared to scenario 1 to reflect the likely congestion development.

Based on the given data, the fuzzy incident delay, as approximated using five levels of presumption with $\alpha = \{0; 0.2; 0.4; 0.6; 0.8; 1.0\}$, can be calculated for further analysis. Fig. 3 shows the predicted fuzzy incident delays under the two given scenarios for vehicles at each CMS. The arrival times of the vehicles at a given CMS were generated based on the distance from the CMS to the incident location and a speed of 100km/h. The following findings are observed from the predicted delay values shown in Figure 8:

- There exists a significant amount of uncertainty in incident delay. This indicates the need to recognize it explicitly in information provision. For example, instead of displaying a single estimate of delay on a CMS, an interval of possible delay, such as ‘incident delay between 15–20 min’, should be used.
- CMS at different locations (distances from an incident spot) should display different delay information to account for differences in vehicles’ expected arrival time. Generally, during the time period that incident congestion starts to build up (scenario 1), CMS farther

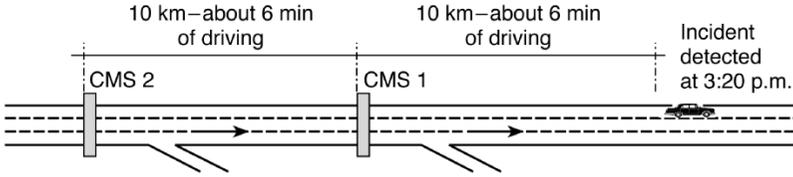


FIGURE 7 Freeway section with an incident.

away from the incident spot (CMS 2) should display delay values higher than those on CMS nearer to the incident spot (CMS 1). Conversely, when the incident is soon to be removed (scenario 2), CMS near the incident spot (CMS 1) should display delay values

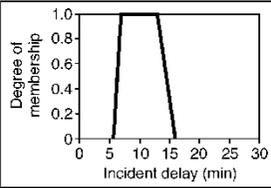
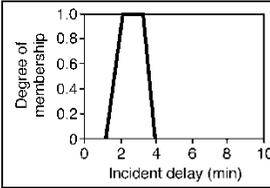
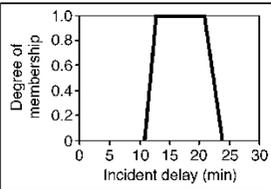
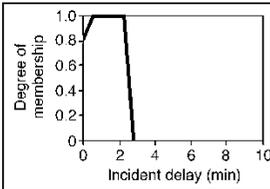
		Scenario 1	Scenario 2
Prediction time:		3:20 p.m.	3:40 p.m.
Current queue, Q (pcu):		{14,18,22,26}	{52,56,64,68}
Incident duration, L (min)		{25,29,31,35}	{4,5,5,5,5}
Traffic volume, V (pch/h):		{3590,3610,4390,4410}	{3590,3610,4390,4410}
Capacity with incident, C (pcu/h):		{1430,1450,1750,1770}	{1430,1450,1750,1770}
Full capacity (pcu/h):		5400	5400
CMS 1	Arrival time, T (min)	{4,9,7,1,5,9,6,1}	{4,9,7,1,5,9,6,1}
	Predicted delay, D (min)		
	Suggested message	'Incident delay 5–15 min'	'Incident delay 1–4 min'
CMS 2	Arrival time, T (min)	{10,3,11,3,12,7,13,7}	{10,3,11,3,12,7,13,7}
	Predicted delay, D (min)		
	Suggested message	'Incident delay 10–25 min'	'Incident delay under 3 min'

FIGURE 8 Estimation of fuzzy incident delay for CMS.

higher than those on CMS farther away from the incident spot (CMS 2).

5. CONCLUSION

This paper developed a fuzzy queuing model that can be used to predict the delay that a vehicle would experience if traveling through an incident location. In contrast to the traditional deterministic models, the proposed model explicitly considers the uncertainties involved in future traffic arrivals, incident duration, departure rate during incident and current queue status and allows easy incorporation of imprecise and vague information on these variables. The model developed in this paper does not require significant additional data or computational requirements over traditional methods and therefore may be readily adopted for ATMS/ATIS applications or simulation studies.

The numerical example has shown that that a deterministic model may overestimate and underestimate the expected incident delay, depending on when the vehicle arrives at the incident location. The maximum estimation error is found to be highly sensitive to the fuzziness of traffic arrival rate, incident duration and capacity during incident. This implies that it should be cautious to use a deterministic model for predicting the expected incident delay when some of the input variables are subject to large variation and imprecision.

It should be pointed out that the methodology presented in this paper assumes that the input variables can be modeled as fuzzy numbers and the related membership functions are known a priori. As a result, the implementation of the proposed model requires an interface to generate membership functions of the input variables based on various sources of information in real-time. The next step of this research will focus on the development of this type of interface and the calibration and refinement of the proposed model for application in ATIS/ATMS.

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